Homework 7: Multivariate Optimization

CS 205A: Mathematical Methods for Robotics, Vision, and Graphics (Winter 2018)
Stanford University
Due Thursday, Mar 8, before 11:59 PM (via gradescope)

Textbook problems:

1. 9.1 (10 points). Convexity of $||Ax - b||$
2. 9.12 (20 points). BFGS Symmetric Rank 1 Update
3. 10.12 (25 points). Modified Gradient Descent.

Julia Programming Assignment 45 points):
In this problem you will use Gradient Descent and Newton's method to minimize two functions:

A The Rosenbrock function
$$R(x, y) = (a - x)^2 + b(y - x^2)^2$$
For our problem, $a = 1$, $b = 100$. Our initial condition for this problem is $x = 0, y = 0$.

B The potential energy of a system of 1-dimensional springs with n degrees of freedom and fixed endpoints.
$$PE_{springs} = \frac{1}{2}k_0(|x_1 - x_L| - d_0)^2 + \sum_{i=1}^{n-1} \frac{1}{2}k_i(|x_{i+1} - x_i| - d_i)^2 + \frac{1}{2}k_n(|x_R - x_n| - d_n)^2$$
The ith spring is connected between x_i and x_{i+1}, has a rest length of d_i, and k_i is its spring constant. For this problem, $d_i = 1 \forall i$, and $n = 10$. x_L is a fixed anchor point on the left side of the springs that x_1 is connected to. x_R is a fixed anchor point on the right side that connects to x_n. There are $n + 1$ springs, and n degrees of freedom. Spring constants alternate between 1 and 0.5 as seen in the starter code. Note: We require that $x_{i+1} \geq x_i \forall i$ and $x_L < x_1$ and $x_n < x_R$

In this problem you will implement Gradient Descent and Newton's Method. The Gradient Descent line search should be implemented using the 1D Newton's method found in Section 9.3 of the textbook. Your methods may be correct but fail on certain problems. If a method fails (for instance, the line search cannot find a $t > 0$), then exit the function and report the current estimate of the optimum x^*. For all algorithms you implement, let your maximum number of iterations by 10000 and the epsilon you use to be $1e - 8$.

1. Run both methods on the Rosenbrock function. Report the minima that each method arrives at. Report the number of iterations required by both methods, or the largest number of steps required until failure.

2. Run both methods to optimize $PE_{springs}$ when $x_L = 0$ and $x_R = 11$. Report the minima that each method arrives at. Report the number of iterations required by both methods, or the largest number of steps required until failure. In this part, the initial condition is $x0 = linspace(1, x_R - 1, n) * 0.9$

3. Run both methods to optimize $PE_{springs}$ when $x_L = 0$ and $x_R = 5$. In this part, the initial condition is $x0 = linspace(1, x_R - 1, n)$

   (a) Report the minima that each method arrives at. Report the number of iterations required by both methods, or the largest number of steps required until failure. Do both methods produce a physical optimum where $x_i < x_{i+1} \forall i$?

   (b) What is the smallest integer value of x_R such that Newton's method provides a valid optimum? Start with $x_R = 10$ and decrement by 1 until you find the breaking point. Initial Condition: $x0 = linspace(1, x_R - 1, n)$ (recompute for each new x_R value).

   (c) What is the smallest integer value of x_R such that gradient descent provides a valid optimum? Start with $x_R = 10$ and decrement by 1 until you find the breaking point. Initial Condition: $x0 = linspace(1, x_R - 1, n)$ (recompute for each new x_R value).

   Compression Test:

4. In this part, you will use the solution of a larger interval as an initial condition on a smaller interval. Specifically, run Newton's method on $PE_{springs}$ starting at $x_R = 11$ as in part 1. Use this optimization result with the previous value of x_R as the initial condition for the next step. Update the right wall $x_R^{next} = x_R^{prev} - 0.1$. Solve for the optimum over this new interval using Newton's method, and repeat this procedure. Using this iterative compression technique, what is the smallest value of that x_R still produces a valid optimum?

Use the provided starter code for implementations of the Rosenbrock and Spring Energy functions, as well as inital conditions and key parameters.
To simplify submission to GradeScope with your other written homework, export a PDF of a clearly documented Julia Notebook that shows your work.