Midterm
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Big Ideas
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Techniques
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Advice
○

Linear Algebra Review

CS 205A:
Mathematical Methods for Robotics, Vision, and Graphics

Doug James (and Justin Solomon)

Midterm Exam

Tuesday Feb 13, in class

▶ Covers linalg (up to and incl SVD).

▶ Closed book exam.

▶ Can use 1 page of notes (two-sided, human readable)

▶ No calculators or computers allowed (or needed)

▶ SCPD: Arrange proctor for exam at similar time.

▶ Previous midterm exams on website (similar material)

▶ Simple HW this week – short question.

What Have We Done?

$$A\vec{x} = \vec{b}$$

Gaussian Elimination

▶ Codifies the typical approach taken on paper

▶ **Phases:** Forward substitution, back substitution (pivoting)

▶ **Elimination matrices:** Notational convenience, algorithmically *slow*!

LU Factorization

▶ $O(n^3)$ time to compute

▶ Allows for solving linear systems via forward/backward substitution ($O(n^2)$ time)

▶ Might not exist – need pivots (e.g. LUP)

Cholesky Factorization: $LL^\top$

For symmetric, positive definite matrices

QR Factorization

▶ R is **upper triangular**

▶ Q has **orthonormal columns**

▶ **Many algorithms:**
Gram-Schmidt, Householder, Givens

▶ **Least-squares** w/o squaring condition #

Diagonalizability: $D = X^{-1}AX$

▶ Diagonalizable iff there is a full eigenspace

▶ **Spectral theorem:** symmetric/Hermitian $\implies$ full, orthogonal eigenbasis

▶ **Computation:** Variations of power method

▶ **Note:** $AX = XD$ (usually $AX \neq DX$!!)

Singular Value Decomposition

$$A = U\Sigma V^\top$$

Variational Approach

Define energy measuring something desirable and minimize it.

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Lagrange multipliers!

Regularization

Improves conditioning of ill-posed problems

E.g., Tikhonov regularization.

$$E(\vec{x}) = \|A\vec{x} - \vec{b}\|_2^2 + \alpha\|\vec{x}\|_2^2$$

Multiple formulations.
Connection to truncated SVD.

Look for Special Structure

▶ Symmetric

▶ Positive definite

▶ Sparse

▶ Normal equations

▶ Square

▶ Full rank

▶ Block

▶ Triangular

Reduce to Known Algorithm

Show that a specific problem is equivalent to:

▶ Least squares (curve fitting)

▶ Eigenvectors (ODEs, embedding)

▶ Factorization (metric learning)

▶ SVD (principal components analysis)

Stability and Conditioning

Complement algorithmic
analysis with understanding
quality of output

Julia

Not on written midterm, but useful for study.

Advice

Draw matrix pictures.

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Draw matrix pictures.

Experiment.

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Draw matrix pictures.

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Ask for help.

▸ Next